Control Of Non-Linear Vibration Using An Iterative Sherman-Morrison Receptance Method

UNIVERSITY OF Southampton
Institute of Sound and Vibration Research

L. I. Wilmshurst*, M. Ghandchi-Tehrani, S. J. Elliott

1: INTRODUCTION

The Sherman-Morrison receptance method proposed by Ram and Mottershead [1] provides a means of reassigning the poles of a dynamic system using active velocity and displacement feedback control, such that the behaviour of the system is more desirable. The primary advantage of this method is that the poles are assigned using measured receptances instead on a FE model, and so the uncertainties, assumptions and errors associated with FE models are eliminated. This approach is well established for linear systems, and the aim of the research is to extend the method to non-linear systems, using a Duffing oscillator as an example.

2: SHERMAN-MORRISON RECEPTANCE METHOD

The application of feedback control to a linear system can be described as a rank-1 update to the open-loop dynamic stiffness matrix $\mathbf{H}^{-1}(\mathbf{s})$:

$$\hat{\mathbf{H}}(\mathbf{s}) = \left[\mathbf{H}^{-1}(\mathbf{s}) + \mathbf{b}(s\mathbf{f} + \mathbf{g})\right]^{-1} \quad (1)$$

where:

- $\mathbf{H}(\mathbf{s})$ open-loop receptance
- $\hat{\mathbf{H}}(\mathbf{s})$ closed-loop receptance
- $\mathbf{b}$ control distribution vector
- $\mathbf{f}$ velocity control gain vector
- $\mathbf{g}$ displacement control gain vector

Applying the Sherman-Morrison formula to (1) yields:

$$\hat{\mathbf{H}}(\mathbf{s}) = \mathbf{H}(\mathbf{s}) - \frac{\mathbf{H}(\mathbf{s})\mathbf{b}(s\mathbf{f} + \mathbf{g})\mathbf{H}(\mathbf{s})}{1 + (s\mathbf{f} + \mathbf{g})\mathbf{H}(\mathbf{s})\mathbf{b}} \quad (2)$$

The closed-loop poles μ_j are defined where the denominator of the second term in (2) is zero:

$$1 + (\mu_j\mathbf{f} + \mathbf{g})\mathbf{H}(\mu_j)\mathbf{b} = 0 \quad (3)$$

If the closed-loop poles are pre-allocated, then the control gains required to assign the poles are obtained by rearranging (3):

$$\begin{bmatrix}\mathbf{f}\\ \mathbf{g}\end{bmatrix} = \begin{bmatrix}\mu_1\mathbf{H}(\mu_1)\mathbf{b} & \mathbf{H}(\mu_1)\mathbf{b}\\ \cdots & \cdots\\ \mu_{2n}\mathbf{H}(\mu_{2n})\mathbf{b} & \mathbf{H}(\mu_{2n})\mathbf{b}\end{bmatrix}^{-1}\begin{bmatrix}1\\ \cdots\\ 1\end{bmatrix} \quad (4)$$

Experimentally, $\mathbf{H}(\mu_j)$ is obtained from the measured receptance $\mathbf{H}(\omega)$ by applying rational polynomial fractions to curve-fit the measured data.

3: NON-LINEAR CHARACTERISTICS

Non-linear systems differ from linear systems in three primary areas:

- The superposition principle is violated
- The receptances are inhomogeneous (amplitude-dependent) and may undergo distortion
- The system will exhibit a response at harmonics of the forcing frequency.

It is necessary to assume a <u>constant excitation (or displacement) amplitude</u>.

The response at the fundamental forcing frequency is given by the first-order receptance $\mathbf{\Lambda_1}(\omega)$.

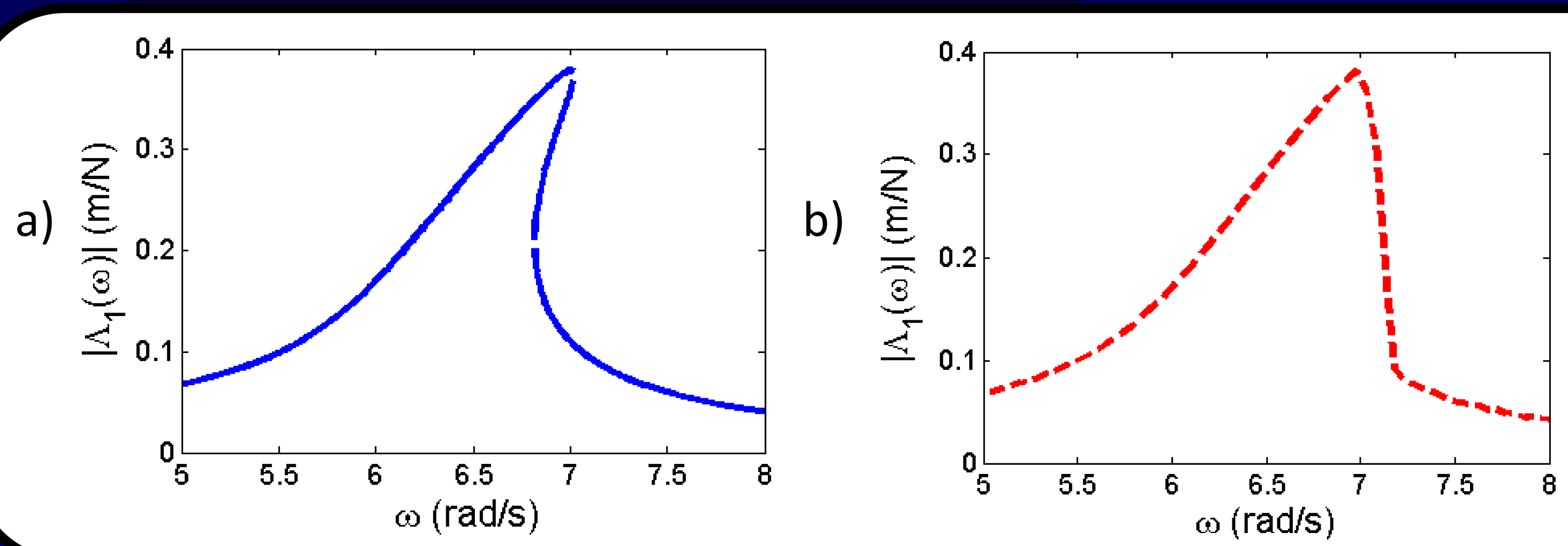

Figure 1: Distorted receptances for a non-linear Duffing oscillator, where a) is theoretical and b) is what would be observed experimentally using an increasing stepped-sine approach.

4: ITERATIVE METHOD

Assumption: The Sherman-Morrison receptance method can be extended to non-linear systems by replacing $\mathbf{H}(\mu_j)$ with the first-order open-loop receptance $\mathbf{\Lambda_1}(\mu_j)$ of the non-linear system.

Problem: The feedback control affects the displacement amplitude of the non-linear system, thereby altering $\mathbf{\Lambda_1}(\mu_j)$.

Solution: Account for alteration by applying a linear modification $\mathbf{P}(\mu_j)$ to $\mathbf{\Lambda_1}(\mu_j)$ and iteratively updating the control gains using the modified receptance $\tilde{\mathbf{\Lambda}}_\mathbf{1}(\mu_j)$ in (4)

$$\mathbf{Q}(\mu_j)_k = \mathbf{N}_{c,k} - \mathbf{N}_o, \qquad \mathbf{P}(\mu_j)_k = -\mathbf{\Lambda_1}(\mu_j)\left[\mathbf{Q}^{-1}(\mu_j)_k + \mathbf{\Lambda_1}(\mu_j)\right]^{-1}\mathbf{\Lambda_1}(\mu_j)$$

$$\tilde{\mathbf{\Lambda}}_\mathbf{1}(\mu_j)_{k+1} = \mathbf{\Lambda_1}(\mu_j) + \mathbf{P}(\mu_j)_k$$

where

- $\mathbf{N_o}$ open-loop nonlinearity
- $\mathbf{N}_{c,k}$ closed-loop nonlinearity for iteration k

$\mathbf{N}_o$ and $\mathbf{N}_{c,k}$ are obtained using a harmonic balance method (HBM) or a Volterra model:

- **HMB:** Assigns the poles accurately, but is not very versatile
- **Volterra:** More generalised, but suffers from a lack of accuracy for strong nonlinearities.

5: NUMERICAL EXAMPLES

The dynamics of a Duffing oscillator subjected to sinusoidal excitation are determined by:

$$m\ddot{y}(t) + c\dot{y}(t) + k_1y(t) + k_3y^3(t) = X\sin(\omega t + \varphi)$$

where m, c, k_1 are the mass, damping, and linear stiffness terms respectively and k_3 is the non-linear cubic stiffness term:

Example 1: Weakly Non-Linear System

$m = 1,\ c = 0.1,\ k_1 = 20,\ k_3 = 0.2,\ X = 1$

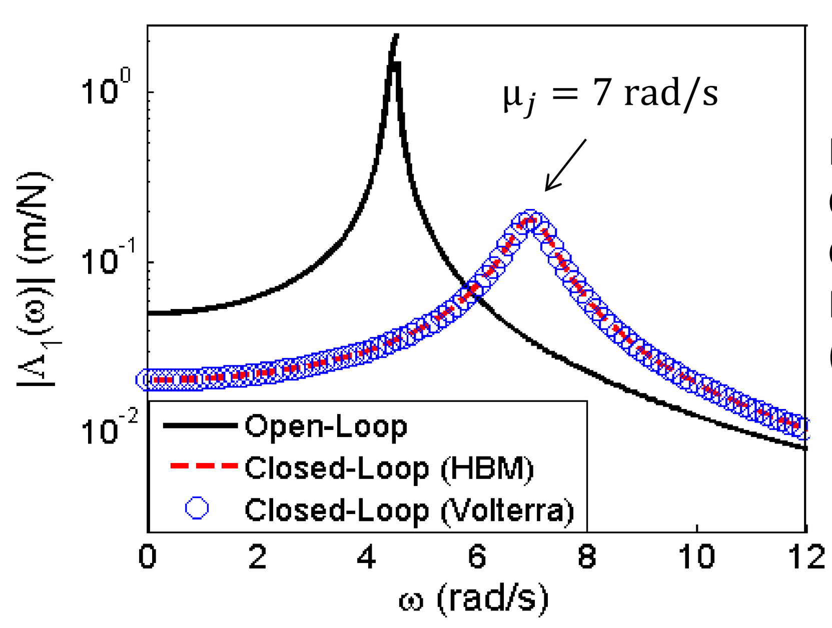

Figure 2: Open-Loop and Closed-Loop Receptances (Example 1)

μ_j	$\mathbf{f},\mathbf{g}$ (HBM)	(Volterra)	μ_j (HBM)	(Volterra)
$-0.15 \pm 7i$	$0.7, 29.16$	$0.7, 29.16$	$-0.15 \pm 7i$	$-0.15 \pm 7i$

Table 1: Numeric values (Example 1).

Example 2: Strongly Non-Linear System

$m = 1,\ c = 0.1,\ k_1 = 20,\ k_3 = 0.8,\ X = 20$

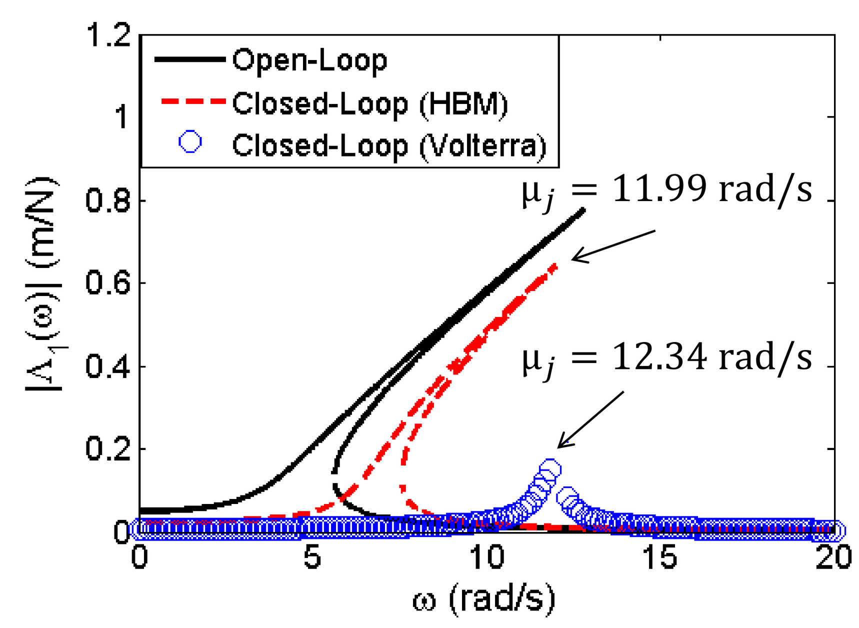

Figure 3: Open-Loop and Closed-Loop Receptances (Example 2)

μ_j	$\mathbf{f},\mathbf{g}$ (HBM)	(Volterra)	μ_j (HBM)	(Volterra)
$-0.065 \pm 12i$	$0.03, 25.39$	$0.25, 119.59$	$-0.065 \pm 11.99i$	$-0.17 \pm 12.34i$

Table 2: Numeric values (Example 2).

6: CONCLUSIONS AND FUTURE WORK

<u>Conclusions</u>

- Poles can be accurately assigned using HBM
- Volterra model can be used for weak nonlinearities

<u>Future Work</u>

- Investigate NARX models
- Test theory with MDOF Duffing oscillator
- Experimental validation using saturation.

REFERENCES

[1] Y. M. Ram and J. E. Mottershead. 2007. Receptance Method in Active Vibration Control. *AIAA Journal*. **45**(3) p. 562-567

*Contact: lw5e10@soton.ac.uk